# SCHOOL DISTRICT OF THE CHATHAMS

## Geometry A Grades 9-12 Full Year

## **Course Overview**

This course provides an understanding of the practical uses of geometry. Students will develop the skill of creating effective arguments through traditional topics of Euclidean geometry, which include, but are not limited to, studies of similarity, congruence, circles, area, and volume. Students will also use constructions to visualize relationships between and among angles, segments, lines, and shapes. This course will reinforce pre-algebra concepts as students continue to develop their problem-solving skills and apply geometry understandings to real world applications.

# New Jersey Student Learning Standards

The New Jersey Student Learning Standards (NJSLS) can be located at <u>www.nj.gov/education/cccs/2020/</u>.

## **Geometry - Congruence:**

G.CO.A.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc. G.CO.A.2 Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).

G.CO.A.3 Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.

G.CO.A.4 Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

G.CO.A.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

G.CO.B.6 Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.

G.CO.B.7 Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. G.CO.B.8 Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

G.CO.C.9 Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.

G.CO.C.10 Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.

G.CO.C.11 Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.

# Geometry - Similarity, Right Triangles, and Trigonometry:

G.SRT.A.1 Verify experimentally the properties of dilations given by a center and a scale factor G.SRT.A.2. Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

G.SRT.A.3 Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

G.SRT.B.4 Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.

G.SRT.B.5 Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

G.SRT.C.6 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.

G.SRT.C.7 Explain and use the relationship between the sine and cosine of complementary angles. G.SRT.C.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

G.SRT.D.9 (+) Derive the formula A = 1/2 ab sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.

G.SRT.D.10 (+) Prove the Laws of Sines and Cosines and use them to solve problems.

G.SRT.D.11 (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

# Geometry - Circles:

G.C.A.1 Prove that all circles are similar.

G.C.A.2 Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle. G.C.A.3 Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.

G.C.A.4 (+) Construct a tangent line from a point outside a given circle to the circle.

G.C.B.5 Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

Geometry - Expressing Geometric Properties with Equations:

G.GPE.A.1 Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

G.GPE.A.2 Derive the equation of a parabola given a focus and directrix.

G.GPE.B.4 Use coordinates to prove simple geometric theorems algebraically.

G.GPE.B.5 Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems.

G.GPE.B.6. Find the point on a directed line segment between two given points that partitions the segment in a given ratio.

G.GPE.B.7 Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.

# Geometry - Geometric Measurement and Dimension:

G.GMD.A.1 Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments.

G.GMD.A.3 Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. G.GMD.B.4 Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

# Geometry - Modeling with Geometry:

G.MG.A.1 Use geometric shapes, their measures, and their properties to describe objects. G.MG.A.2 Apply concepts of density based on area and volume in modeling situations. G.MG.A.3 Apply geometric methods to solve design problems.

(+) Additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics.

# **Technology Standards**

9.4.12.TL.4: Collaborate in online learning communities or social networks or virtual worlds to analyze and propose a resolution to a real-world problem.

# 21st Century Integration | NJSLS 9

9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas 9.4.12.CT.2: Explain the potential benefits of collaborating to enhance critical thinking and problem solving

# **Career Ready Practices**

CRP2. Apply appropriate academic and technical skills.

CRP4. Communicate clearly and effectively and with reason.

CRP6. Demonstrate creativity and innovation.

CRP7. Employ valid and reliable research strategies.

CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.

CRP11. Use technology to enhance productivity.

# **Interdisciplinary Connections**

English Language Arts:

- RST.9-10.4. Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 9-10 texts and topics.
- RST.9-10.7. Translate quantitative or technical information expressed in words in a text into visual form (e.g., a table or chart) and translate information expressed visually or mathematically (e.g., in an equation) into words.

# Science:

• HS-ETS1-2 Design a solution to a complex real-world problem by breaking it down into smaller, more manageable problems that can be solved through engineering.

# Units of Study

Unit 1: Basics of Geometry (~16 days)

- How can we use dynamic geometry software to visualize geometric concepts?
- How can we measure and construct a line segment?
- How can we find the midpoint and length of a line segment in a coordinate plane?
- How can we measure and classify an angle?
- How can we describe angle pair relationships and use these descriptions to find angle measures?

Unit 2: Reasoning and Proofs (~12 days)

- How can we understand and write conditional statements?
- How can we use inductive and deductive reasoning?

Unit 3: Parallel and Perpendicular Lines (~17 days)

- What relationships exist between lines, planes, and pairs of angles?
- What are the relationships between angles formed by parallel lines and transversals?
- How can we use theorems about identifying parallel lines?
- How can we use theorems about perpendicular lines?
- How can we partition a directed line segment and understand slopes of parallel and perpendicular lines?

Unit 4: Transformations (~15 days)

- What are the various forms of transformations?
- How can we identify transformations graphically?
- How can we display translations, reflections, rotations, and dilations?
- Can we recognize congruences and similarity transformations?

Unit 5: Congruent Triangles (~14 days)

- How can we prove and use theorems about angles of a triangle?
- What is congruence in terms of rigid motions?
- How do we prove and use the Side-Angle-Side Congruence Theorem?
- How do we prove and use theorems about isosceles and equilateral triangles?
- How do we prove and use the Side-Side-Side Congruence Theorem?
- How do we prove and use the Angle-Side-Angle Congruence Theorem and the Angle-Angle-Side Congruence Theorem?
- What strategies are useful when using congruent triangles in proofs and to measure distances?
- How can coordinates be utilized to write proofs?

Unit 6: Relationships within Triangles (~16 days)

- How can we use theorems about perpendicular and angle bisectors?
- What is the median and altitude of a triangle?
- Can we find and use the midsegments of a triangle?
- How can we write indirect proofs and understand inequalities in a triangle?
- How can I understand the relationship between two triangles?

Unit 7: Quadrilaterals and Polygons (~16 days)

- How do you find the angle measures of polygons?
- How can we prove and use properties of parallelograms?
- How can we prove that a quadrilateral is a parallelogram?

- Are we able to explain the properties of special parallelograms?
- How can the properties of trapezoids and kites be used to find measures?

Unit 8: Similarity (~16 days)

- How can we utilize similarity statements?
- What are the strategies when determining if two polygons are similar?
- Are we able to understand and apply similarity theorems?
- Can we utilize proportional relationships mathematically with similar figures?

Unit 9: Right Triangles and Trigonometry (~18 days)

- How can we understand and apply the Pythagorean theorem?
- How can we understand and use special right triangles?
- How are proportional relationships applied to right triangles?
- What are the trigonometric ratios?
- How can we find unknown side lengths and angle measures of right triangles?

# Unit 10: Circles (~16 days)

- How can we identify lines and segments that intersect circles and use them to solve problems?
- What are the properties of circles and their measures?
- How can we apply various circular theorems?
- What is the form for an equation of a circle?
- What steps are needed to graph and write equations of parabolas?

Unit 11: Circumference and Area (~12 days)

- What are circumference, arc length, and radian measure?
- How can we find areas of circles and areas of sectors of circles?
- How can we find angle measures and areas of regular polygons?
- Are we able to understand the concept of population density and modeling with area?

Unit 12: Surface Area and Volume (~12 days)

- How can we describe and draw cross sections?
- What strategies are useful when finding volume of 3-D figures?
- Are we able to understand the concept of density and modeling with volume?
- How can we sketch and use solids of revolution?

# Learning Objectives/Discipline Standards of Practice

## Learning Objectives:

- Name points, lines, and planes.
- Name segments and rays.
- Sketch intersections of lines and planes.
- Solve real-life problems involving lines and planes
- Use the Ruler Postulate.
- Copy segments and compare segments for congruence.
- Use the Segment Addition Postulate
- Find segment lengths using midpoints and segment bisectors.
- Use the Midpoint Formula.
- Use the Distance Formula.
- Find perimeters and areas of polygons in the coordinate plane.

- Name angles.
- Measure and classify angles.
- Identify congruent angles.
- Use the Angle Addition Postulate to find angle measures.
- Bisect angles
- Identify complementary and supplementary angles.
- Identify linear pairs and vertical angles.
- Write conditional statements.
- Write biconditional statements.
- Determine if conditional statements are true by using truth tables.
- Use inductive reasoning to make conjectures
- Use deductive reasoning to verify conjectures
- Distinguish between inductive and deductive reasoning
- Identify postulates represented by diagrams.
- Sketch a diagram given a verbal description
- Interpret a diagram.
- Identify algebraic properties of equality
- Use algebraic properties of equality to solve equations.
- Use properties of equality to solve for geometric measures.
- Explain the structure of a two-column proof.
- Write a two-column proof.
- Identify properties of congruence
- Prove geometric relationships by writing flowchart proofs
- Prove geometric relationships by writing paragraph proofs.
- Identify lines and planes
- Identify parallel and perpendicular lines
- Identify pairs of angles formed by transversals
- Use properties of parallel lines to find angle measures.
- Use theorems about parallel lines.
- Use theorems to identify parallel lines.
- Construct parallel lines.
- Find the distance from a point to a line
- Construct perpendicular lines perpendicular bisectors
- Use theorems about perpendicular lines.
- Partition directed line segments using slope.
- Use slopes to identify parallel and perpendicular lines.
- Write equations of parallel and perpendicular lines.
- Find the distance from a point to a line.
- Translate figures.
- Write a translation rule for a given translation.
- Explain what a rigid motion is.
- Perform a composition of translations on a figure.
- Reflect figures.
- Perform compositions with reflections.
- Identify line symmetry in polygons
- Rotate figures.
- Perform compositions with rotations.
- Identify rotational symmetry in polygons.
- Identify congruent figures.
- Describe congruence transformations.

- Use congruence transformations to solve problems.
- Identify dilations.
- Dilate figures.
- Solve real-life problems involving scale factors and dilations.
- Perform similarity transformations.
- Describe similarity transformations.
- Prove that figures are similar.
- Classify triangles by sides and by angles
- Prove theorems about angles of triangles
- Find interior and exterior angle measures of triangles
- Use rigid motions to show that two triangles are congruent
- Identify corresponding parts of congruent polygons
- Use congruent polygons to solve problems
- Use rigid motions to prove the SAS Congruence Theorem
- Use the SAS Congruence Theorem
- Prove and use theorems about isosceles triangles
- Prove and use theorems about equilateral triangles
- Use rigid motions to prove the SSS Congruence Theorem
- Use the SSS Congruence Theorem
- Use the Hypotenuse-Leg Congruence Theorem
- Use rigid motions to prove the ASA Congruence Theorem
- Prove the AAS Congruence Theorem
- Use the ASA and AAS Congruence Theorems
- Use congruent triangles to prove statements
- Use congruent triangles to solve real-life problems
- Use congruent triangles to prove constructions
- Place figures in a coordinate plane
- Write plans for coordinate proofs
- Write coordinate proofs
- Identify a perpendicular and angle bisector
- Use theorems about bisectors to find measures in figures
- Write equations of perpendicular bisectors
- Find the circumcenter and incenter
- Circumscribe a circle about a triangle
- Inscribe a circle within a triangle
- Use points of concurrency to solve real life problems
- Draw medians and altitudes of triangles.
- Find the centroid of a triangle.
- Find the orthocenter of a triangle.
- Use the midsegment of triangles in the coordinate place to solve problems
- Solve real-life problems involving midsegments.
- Order the angles of a triangle given the side lengths
- Order the side lengths given the angles.
- Determine possible side lengths of triangles.
- Explain the hinge theorem
- Compare measures of triangles.
- Solve real-life problems using hinge theorem.
- I can find the sum of the interior angle measures of a polygon. I can find interior angle measures of polygons.
- I can find interior angle measures of polygons.

- I can find exterior angle measures of polygons.
- Prove properties of parallelograms.
- Use properties of parallelograms.
- Solve problems involving parallelograms in the coordinate plane.
- Identify features of a parallelogram.
- Prove that a quadrilateral is a parallelogram.
- Find missing lengths that make a quadrilateral a parallelogram.
- Show that a quadrilateral in the coordinate plane is a parallelogram.
- Identify special quadrilaterals.
- Explain how special parallelograms are related.
- Find missing measures of special parallelograms.
- Identify special parallelograms in a coordinate plane.
- Identify trapezoids and kites.
- Use properties of trapezoids and kites to solve problems.
- Find the length of the midsegment of a trapezoid.
- Explain the hierarchy of quadrilaterals.
- Use similarity statements.
- Find corresponding lengths in similar polygons.
- Find perimeters and areas of similar polygons.
- Decide whether polygons are similar.
- Use similarity transformations to prove the Angle-Angle Similarity Theorem.
- Use angle measures of triangles to determine whether triangles are similar.
- Prove triangle similarity using the Angle-Angle Similarity Theorem.
- Solve real-life problems using similar triangles.
- Use the SSS and SAS similarity theorems to determine whether triangles are similar.
- Use similar triangles to prove theorems about slopes of parallel and perpendicular lines.
- Use proportionality theorems to find lengths in triangles.
- Find lengths when two transversals intersect three parallel lines.
- Find lengths when a ray bisects an angle of a triangle.
- List common Pythagorean triples.
- Find missing side lengths of right triangles.
- Classify a triangle as acute, right, and obtuse given side lengths.
- Find side lengths in 45° 45° 90° triangles
- Find side lengths in 30° 60° 90° triangles.
- Use special right triangles to solve real-life problems.
- Explain the right triangle similarity theorem
- Find the geometric mean of two numbers
- Find the missing dimensions in right triangles.
- Explain the tangent ratio
- Find the tangent ratio.
- Use the tangent ratio to solve real-life problems.
- Explain the sine and cosine ratio.
- Find the sine and cosine ratios.
- Use sine and cosine ratios to solve real-life problems.
- Explain inverse trigonometric ratios
- Use inverse trigonometric ratios to approximate angle measures
- Solve right triangles.
- Solve real-life problems by solving right triangles.
- Find areas of triangles using formulas that involve sine.
- Solve triangles using the Law of Sines.

- Solve triangles using the Law of Cosines.
- Identify special segments and lines that intersect circles.
- Draw and identify common tangents
- Use properties of tangents to solve problems
- Find arc measures
- Identify congruent arcs
- Prove that all circles are similar
- Use chords of circles to find arc measures.
- Use chords of circles to find lengths
- Describe the relationship between a diameter and a chord perpendicular to a diameter
- Find the center of a circle given three points on the circle
- Find measures of inscribed angles and intercepted arcs
- Find angle measures of inscribed polygons
- Construct a square inscribed in a circle
- Identify angles and arcs determined by chords, secants, and tangents
- Find angle measures and arc measures involving chords, secants and tangents.
- Use circumscribed angles to solve problems.
- Find lengths of segments of chords.
- Identify segments of secants and tangents.
- Find lengths of segments of secants and tangents
- Write equations of circles.
- Find the center and radius of a circle
- Graph equations of circles
- Write coordinate proofs involving circles
- Explain the relationships among the focus, the directrix, and the graph of a parabola.
- Graph parabolas
- Write equations of parabolas
- Use the formula for the circumference of a circle to find measures.
- Find arc lengths and use arc lengths to find measures.
- Solve real-life problems involving circumference.
- Explain radian measure and convert between degree and radian measure.
- Use the formula for the area of a circle to find measures.
- Find areas of sectors of circles.
- Solve problems involving areas of sectors.
- Find areas of rhombuses and kites.
- Find angle measures in regular polygons.
- Find areas of regular polygons.
- Explain how the area of a triangle is related to the area formulas for rhombuses, kites, and regular polygons.
- Explain what population density means.
- Find and use population densities.
- Use area formulas to solve problems
- Describe attributes of solids
- Describe and draw cross sections.
- Solve real-life problems involving cross sections.
- Find volumes of prisms and cylinders
- Find surface areas and volumes of similar solids.
- Find volumes of pyramids.
- Use volumes of pyramids to find measures
- Find volumes of similar pyramids.

- Find volumes of composite solids containing pyramids
- Find surface areas of cones.
- Find volumes of cones.
- Find the volumes of similar cones.
- Find the volumes of composite solids containing cones.
- Find surface areas of spheres.
- Find volumes of spheres.
- Find the volumes of composite solids.
- Explain what density means
- Use the formula for density to solve problems.
- Use geometric shapes to model objects
- Solve modeling problems.
- Sketch and describe solids of revolution
- Find surface areas and volumes of solids of revolution
- Form solids of revolution in the coordinate plane.

## **Discipline Standards of Practice:**

- MP.1: Make sense of problems and persevere in solving them
- MP.2: Reason abstractly and quantitatively
- MP.3: Construct viable arguments
- MP.4: Model with Mathematics
- MP.5: Use appropriate tools strategically
- MP.6: Attend to precision
- MP.7: Look for and make use of structure
- MP.8: Look for and express regularity in repeated reasoning

# **Instructional Resources and Materials**

Whole class resources have been identified with an asterisk.

## **Resources**\*

Geometry, Larson, Ron, Laurie Boswell, Big Ideas Learning, LLC, 2022.

## **Supplemental Resources\***

- Kuta Software
- TI-Smartview
- Google Suite
- Desmos Classroom Activities
- EdPuzzle

## Materials

- TI-84/83+ Graphing Calculator\*
- Desmos Graphing Calculator
- Graph paper

# **Assessment Strategies**

Assessment is designed to measure a student's mastery of a course standard and learning objective. Assessment can be used for both instructional purposes (formative assessment) and for evaluative purposes (summative assessment). The following is a general list of the many forms assessment may take in learning.

- Tests •
- •
- Quizzes Skill Checks •
- Projects
- Unit Assessments •